$$\begin{cases} u_{tt} = 8(u_{xx} + u_{yy} + u_{zz}) + t^2x^2 \\ u\big|_{t=0} = y^2 \quad u_t\big|_{t=0} = z^2 \end{cases}$$

$u = v + y^2 + tz^2$ (чтобы обнулить нач. условие)

$$\begin{cases} v_{tt} = 8(v_{xx} + v_{yy} + v_{zz}) + 16 + 16t + t^2x^2 \\ v\big|_{t=0} = 0 \quad v_t\big|_{t=0} = 0 \end{cases}$$

от y z ничего не зависит.

$\exists$ $v = x^2T(t) + W(t)$

$x^2T'' + W'' = 16T + 16 + 16t + t^2x^2$

1) $T'' = t^2 \Rightarrow T = \frac{t^4}{3\cdot 4} + At + B$

$x^2T(0) + W(0) \Rightarrow T(0) = 0 \Rightarrow W(0) = 0$

$x^2T'(0) + W'(0) \Rightarrow T'(0) = 0 \Rightarrow W'(0) = 0$

$\Rightarrow T = \frac{t^4}{3\cdot 4}$

2) $W'' = 16\frac{t^4}{3\cdot 4} + 16 + 16t \Rightarrow$

$W = \frac{16\,t^6}{3\cdot 4\cdot 5\cdot 6} + \frac{16t^3}{2\cdot 3} + \frac{16}{2}t^2 + C\cdot t + D$

$W(0) = 0 \quad W'(0) = 0 \Rightarrow C = 0 \quad D = 0$

$\Rightarrow v = \frac{x^2t^4}{12} + \frac{2}{45}t^6 + \frac{8}{3}t^3 + 8t^2$

$$\boxed{u = y^2 + tz^2 + \frac{x^2t^4}{12} + \frac{2}{45}t^6 + \frac{8}{3}t^3 + 8t^2}$$

№ 1 Д/з 7

$$\begin{cases} u_{tt} = a^2 \Delta_2 u(x,y,t) & 0<x<\pi, \quad 0<y<\pi \quad t>0 \\ u(x,y,0) = 0 \\ u_t(x,y,0) = Axy(\pi-x)(\pi-y) \\ u|_{x=0} = u|_{x=\pi} = u|_{y=0} = u|_{y=\pi} = 0 \end{cases}$$

$A = const \neq 0$

$\exists \; u = W(x,y) \cdot T(t)$

$$\frac{\Delta_2 W}{W} = \frac{T''}{a^2 T} = -\lambda$$

$$\Delta_2 W = -\lambda W$$

$\exists \; W = X(x) \cdot Y(y)$

$$X'' Y + X \cdot Y'' + \lambda X \cdot Y = 0$$

$$\frac{X''}{X} + \frac{Y''}{Y} + \lambda = 0$$

$-\mu \qquad -\nu$

$$\begin{cases} X'' + \mu X = 0 \\ X(0) = 0, \quad X(\pi) = 0 \end{cases}$$

$X = A\sin(\sqrt{\mu}x) + B\cos(\sqrt{\mu}x)$

$X(0) = B = 0$

$X(\pi) = A\sin(\sqrt{\mu}\pi) = 0$

$\sqrt{\mu}\pi = \pi n \qquad n \in \mathbb{N}$ ~~$\cup \{0\}$~~

$\mu = n^2, \quad n = 1, 2$

$$\begin{cases} Y'' + \nu Y = 0 \\ Y(0) = 0, \quad Y(\pi) = 0 \end{cases}$$

$Y_k = \sin ky, \quad \nu = k^2$

$X_n = \sin nx, \quad \mu = n^2$

$\lambda = k^2 + n^2$

$T'' + a^2 \lambda T = 0$

$T = C\sin(a\sqrt{\lambda}t) + D\cos(a\sqrt{\lambda}t)$

$$u = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} \sin nx \cdot \sin ky \cdot (C_{kn}\sin(a\sqrt{\lambda_{kn}}t) + D_{kn}\cos(a\sqrt{\lambda_{kn}}t))$$

$$u(x,y,0) = \sum_{k,n=1}^{\infty} \sin nx \cdot \sin ky \cdot D_{kn} = 0$$
$$\Rightarrow D_{kn} = 0$$

$$u_t(x,y,0) = \sum_{k,n=1}^{\infty} \sin nx \cdot \sin ky \cdot C_{kn} \cdot a\sqrt{\lambda_{kn}} = Axy(\pi-x)(\pi-y)$$

$$\overset{a\sqrt{\lambda_{kn}}}{C_{kn}^{\vee}} = \frac{2}{\pi}\cdot\frac{2}{\pi}\int_0^{\pi}\int_0^{\pi} Axy(\pi-x)(\pi-y)\sin nx \cdot \sin ky \, dx\,dy =$$

$$= \frac{4A}{\pi^2}\int_0^{\pi} x(\pi-x)\cdot\sin nx\,dx \cdot \int_0^{\pi} y(\pi-y)\cdot\sin ky\,dy$$

$$\int_0^{\pi} x(\pi-x)\cdot\sin nx\,dx = x(\pi-x)\left(-\frac{\cos nx}{n}\right)\Big|_0^{\pi} - \int_0^{\pi}(\pi-2x)\left(-\frac{\cos nx}{n}\right)dx = (\pi-2x)\left(\frac{\sin nx}{n^2}\right)\Big|_0^{\pi} - \int_0^{\pi}(-2)\frac{\sin nx}{n^2}dx = 0 + 2\frac{-\cos nx}{n^3}\Big|_0^{\pi} = \frac{2}{n^3}(1-(-1)^n)$$

$$\Rightarrow \overset{a\sqrt{\lambda_{kn}}}{C_{kn}^{\vee}} = \frac{4A}{\pi^2}\cdot\frac{2}{n^3}(1-(-1)^n)\cdot\frac{2}{k^3}(1-(-1)^k)$$

$C_{kn} \neq 0$ только когда n и k нечетные

$$\cancel{\overset{a\sqrt{\lambda_{kn}}}{C_{kn}^{\vee}} = \frac{64}{\pi^2(2n+1)^3(2k+1)^3}}$$

$$\Rightarrow \cancel{C_{kn} = \frac{64}{\pi^2 a (2n+1)^3(2k+1)^3\sqrt{k^2+n^2}}}$$

$$C_{2k+1,\,2n+1} = \frac{64A}{\pi^2 (2k+1)^3 (2n+1)^3 \cdot a\sqrt{(2k+1)^2+(2n+1)^2}}$$

$$\Rightarrow \boxed{u = \sum_{n,k=1}^{\infty} \frac{64A}{\pi^2 (2k+1)^3 \cdot (2n+1)^3 \cdot a\sqrt{(2k+1)^2+(2n+1)^2}} \cdot \sin(2n+1)x \cdot \sin(2k+1)y \cdot \sin\left(a\sqrt{(2k+1)^2+(2n+1)^2}\,t\right)}$$ – ответ.

$$\begin{cases} u_{tt} = \Delta_2 u + \sin t \cdot \sin x \cdot \sin y & 0<x<\pi, \quad 0<y<\pi \quad t>0 \\ u(x,y,0) = u_t(x,y,0) = 0 \\ u|_{x=0} = u|_{x=\pi} = u|_{y=0} = u|_{y=\pi} = 0 \end{cases}$$

$$u = v + \sin x \cdot \sin y \cdot \sin t$$

$$\begin{cases} v_{tt} - \sin x \cdot \sin y \cdot \sin t = -2\sin x \cdot \sin y \cdot \sin t + \Delta_2 v + \sin x \cdot \sin y \cdot \sin t \\ v(x,y,0) + 0 = 0 \\ v_t|_{t=0} + \sin x \cdot \sin y \cdot \cos t|_{t=0} = 0 \\ v|_{x=0} = v|_{x=\pi} = v|_{y=0} = v|_{y=\pi} = 0 \end{cases}$$

$$\begin{cases} v_{tt} = \Delta_2 v \\ v(x,y,0) = 0 \\ v_t(x,y,0) = -\sin x \cdot \sin y \\ v|_{x=0} = v|_{x=\pi} = v|_{y=0} = v|_{y=\pi} = 0 \end{cases}$$

в №1 находили $W_{nk} = \sin nx \cdot \sin ky$ $\quad \lambda_{nk} = \sqrt{n^2+k^2}$ для
$T_{nk} = C\sin\sqrt{\lambda}\, t + D \cos(\sqrt{\lambda}\, t)$ этого ур-ия.

$$v = \sum_{n,k=1}^{\infty} \sin nx \cdot \sin ky \cdot (C_{nk} \sin\sqrt{\lambda_{nk}}\, t + D_{nk}\cos\sqrt{\lambda_{nk}}\, t)$$

$v(x,y,0) = 0 \quad \Rightarrow \quad D_{nk} = 0$

$$v_t(x,y,0) = -\sin y \cdot \sin x = \sum_{n,k=1}^{\infty} \sin nx \cdot \sin ky \cdot \sqrt{\lambda_{nk}} \cdot C_{nk}$$

$\Rightarrow \sqrt{\lambda_{11}}\, C_{11} = -1, \quad \Rightarrow \quad C_{11} = -\dfrac{1}{\sqrt{\lambda_{11}}} = -\dfrac{1}{\sqrt{2}}$

$C_{nk} = 0$ – остальные

$\Rightarrow v = \sin x \cdot \sin y \cdot \left(-\dfrac{1}{\sqrt{2}}\right) \cdot \sin\sqrt{2}\, t$

$$\Rightarrow \boxed{u = \sin x \cdot \sin y \cdot \sin t - \sin x \cdot \sin y \, \frac{\sin\sqrt{2}\, t}{\sqrt{2}}}$$